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CONCEPT MAPS AS A TEACHING/LEARNING TOOL IN SECONDARY SCHOOL MATHEMATICS. ANALYSIS OF AN EXPERIENCE

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Abstract: This paper presents an experiment designed to test the different uses of concept maps (CMs), as a tool to promote meaningful learning (ML) in the teaching/learning process for a math topic. The precise aims are to assess its usefulness in the design of an innovative instructional module (IM) on the topic of proportionality, as a learning tool to help students grasp the content of the module, as a means of assessing students' prior knowledge of the topic and monitoring their progress. In a standard schoolroom setting, the implementation of a theoretically grounded IM gave a group of second-grade secondary students at the Ikastola San Fermin School the opportunity to learn about the topic of proportionality in a more meaningful manner. This is demonstrated by statistical analysis of ML indicators, using the SPSS (Statistical Package for the Social Sciences) and an evaluation of the evolution of the students' CMs throughout the instructional module.

1 Introduction

The Second International Conference on Concept Mapping (San José, Costa Rica, 2006) included a special session on Concept Mapping in mathematics, moderated by leading researchers: Nancy R. Romance from Florida Atlantic University (USA), Jean Schmittau from State University of New York at Binghamton (USA) and Karoline Afamasaga-Fuata'i from University of New England (Australia). The proceedings of that session reproduce a number of case-studies illustrating the use of concept maps (CMs) in the teaching of mathematics. These include the use of CMs to help students to grasp the concept of the positional system (Schmittau & Vagliardo, 590-597); the development of a Concept Mapping approach to the teaching of mathematics in secondary schools (Caldwell, Al-Rubaee, Lipkin, Caldwell & Campese, 2006); a study by Pérez Flores (2006); the evaluation of multidimensional CMs (Huerta, 2006); and CMs in the learning stages of Van Hiele's educational model (Esteban Duarte, Vasco Agudelo & Bedoya Beltrán, 2006).

These communications and spaces for the development of more thorough math instruction are mentioned here to highlight the validity and importance of CM's in the area of high school mathematics. A new field is therefore opening up for the use of CM's as an unbeatable tool for the promotion of meaningful learning (ML) and the replacement of rote learning (RL) among students and the detection of certain patterns that can be considered valid learning predictors.

Also presented at the above-mentioned Congress was a study by Pozueta, Guruceaga & González (2006), in which the main objective was to detect ML indicators through the analysis of students' CMs, in a context in which second-grade secondary students worked with proportionality topics from the area of mathematics. In anticipation of the effectiveness of CMs as a tool to promote ML in students, they were used in the design and delivery of the instruction on the topic, and also to assess the knowledge acquired by the students. The results illustrated the effectiveness of CMs in achieving more meaningful learning in the evaluated students.

As noted in the cited work, proportionality is not a simple concept Rapetti (2003) notes the complexity involved in acquiring the notion of proportion and claims that students need to be presented with a range of situations varying in numerical complexity and in the type of magnitudes related, because some students have difficulty when faced with the need to consider quantities in relation to one another, that is, when required to see them in other than absolute terms. This stands in the way of their understanding what they need to learn in order to grasp the notion of proportionality.

When it comes to seeking references for the analysis of proportionality in our teaching/learning context, it is important to take into account the research that has gone into defining the concepts of ratio and/or proportion. Lesh et al. (1988) examine and compare the views of various authors such as Vergnaud, Schwartz and Kaput, among others, regarding the nature of ratios.

Other outstanding contributions include Freudenthal (1983), who highlights the importance of distinguishing between internal (or "within") ratios and external (or "between") ratios. Internal ratios are those that compare different quantities belonging to the same system, and external ratios are those that compare different quantities belonging to different systems. Nesher and Sukenik (1989), in a brief overview of previous research on the concepts of ratio and proportion, mention a common procedure widely used in many studies, which is to administer a test including ratio problems (written or oral and with or without illustrations) and analyse students' answers in terms of the strategies they use to solve the problems. They report that one of the commonest errors in children of various age groups is the use of the additive strategy, whereby children see the relationship between the ratios as the difference between the terms and fail to capture its multiplicative nature.

These theoretical considerations have led to our interest in advancing with the use of CMs in the topic of proportionality, testing its usefulness in various practical aspects, such as the design of an instructional module (IM) for the topic, based on the promotion of positive attitudes, the identification of students' prior notions and conceptually transparent curricular and instructional material on the one hand, and the analysis of the evolution of the students' learning process on the other, all of which will promote meaningful learning in students in the early years of secondary education.

2 Research design and development

This paper sets out to test different uses of CMs as a tool to promote meaningful learning, in other words, to assess their usefulness in tasks such as:

- Designing an innovative IM for the topic of proportionality, consistent with Ausubel's (1976) recommendation to take into account what students already know about the target topic. The maps drawn by the students prior to instruction can be used to identify the starting point of the learning process for each of them. Ausubel also proposes beginning the instruction process by presenting the more inclusive concepts relating to the target topic, before dealing with the more specific concepts. Hence the need to clarify which concepts are to be included in the instruction, what significance they will have, what hierarchical relationships and reconciliations there are between them, and how this frame of reference relates to what students already know. Novak (1998) recommends teachers to create a reference CM on which they should set out all the concepts, both inclusive and specific, relating to the chosen topic, in this case, proportionality. Such a reference CM (see Figure 1) serves to identify the most significant conceptual nodes and informs the design and sequencing of the activities.
- To be used as a learning tool by students to grasp the instructional content.
- To identify students' prior knowledge of the topic and analyse the evolution of their learning process. This will be done by means of a comparative analysis of the maps drawn by the students before and after instruction, following the model presented by Guruceaga & González (2004) and focusing on the presence or absence of certain features (see Table 1). In this way, a student's CM will provide a tool to reveal the degree of ML achieved, or alternatively show that the learning has been less meaningful and more of a rote or mechanical nature. The indicators are shown below:

Table 1. Learning indicators.

Indicators of rote/mechanical learning	Indicators of meaningful learning
• No clear differentiation between concepts and linking phrases; direction of the relationships between concepts not shown	• Clear differentiation between concepts and linking phrases; shows the direction of the relationships between concepts
• A minor number of concepts are used	• Most of concepts are used
• A high frequency of erroneous propositions: illogical conceptual hierarchies	• A decreasing trend in erroneous propositions
• An incorrect hierarchical ordering of concepts in terms of their inclusivity	• There is coherence in the hierarchical organisation of the concepts in terms of their inclusivity
• The most inclusive concepts are not identified	• The most inclusive concept is identified
• Shows long linear relationships, chaining of concepts	• Examples of super-ordination of an inclusive concept • Progressive differentiation between inclusive concepts • Linear relationships between concepts are fewer or totally absent
• Crossed links are few in number and erroneous: a sign of weak integrative reconciliations	• There are numerous crossed links revealing high-level integrative reconciliations

This study was conducted during the 2006-2007 school year in the Pamplona municipal area in a state-aided school (San Fermin Ikastola), where students of all stages, infant, primary, secondary and high school, are taught in the Basque language. The research, which was performed by a highly-experienced secondary mathematics teacher, was structured in various stages:

1. The first step was to create a reference map of the kind mentioned above (see Figure1), pitched to the level of second grade secondary students.

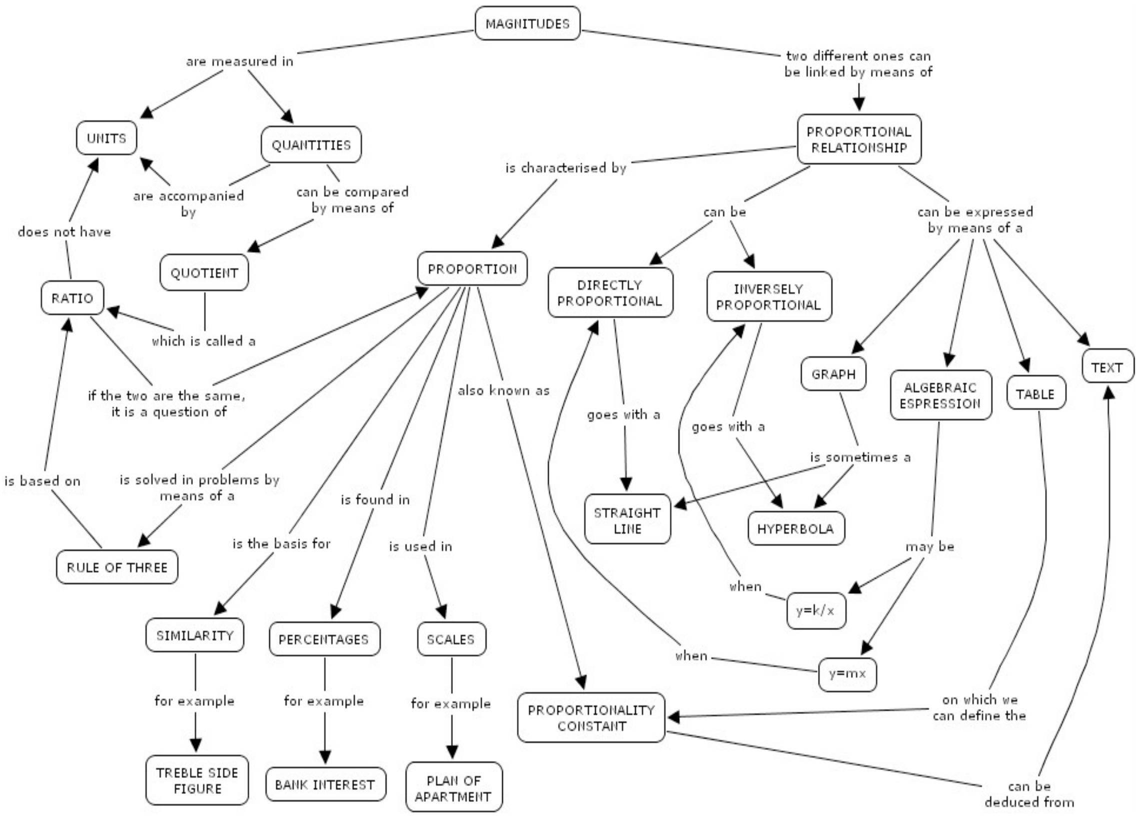

Figure 1. Reference map (Pozueta, 2003).

Twenty-five concepts were selected and the main relationships between them defined according to the teaching aims established in the IM. It should be stressed at this point that the aim of the module was to present and teach proportionality, starting with mathematical situations involving ratio, such as similarity, percentages and scales. The last level in the CM hierarchy therefore shows concrete examples of these situations. The map does not mention the necessary condition of congruence of corresponding angles to define similarity of figures; it refers only to the role of proportional reasoning. The map was also designed to make a clear distinction between ratio defined as a relationship between different quantities of the same magnitude and the proportional relationship that may exist between two different magnitudes, and the various ways in which this proportional relationship can be expressed. Thus, the four ways of expressing a relationship between two magnitudes appear on the right hand side of the map, each labelled to show whether it is a directly proportional or inversely proportional relationship. The definition of ***ratio*** is one that constitutes a violation of Freudenthal's (1983) interpretation, but it is the one that appears in the vast majority of textbooks for the teaching of mathematics in the first grades of secondary education.

2. Three groups of second grade secondary students, a total of 84 individuals, drew a CM prior to instruction. For this they were given the same list of 25 concepts used in the reference map. Figure 2 shows one student's initial map, in which it is possible to observe the few concepts used and the absence of important links.

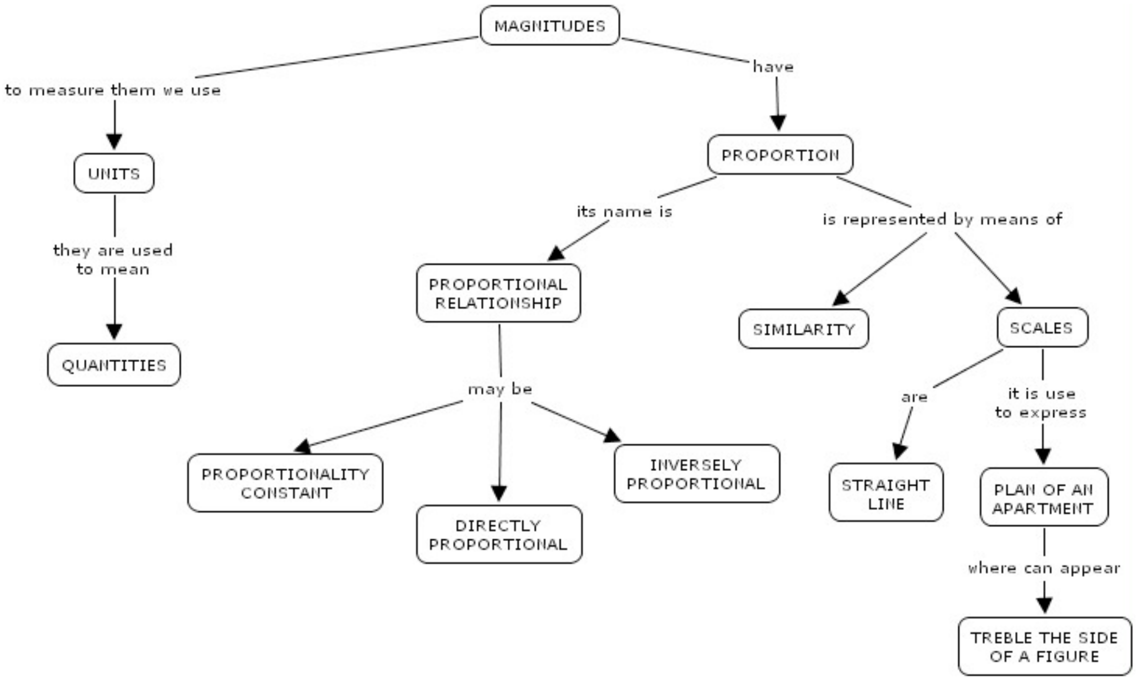

Figure 2. I. A.´s first map.

3. These maps served to reveal the point of departure for each student's learning process and informed the design of the innovative IM on proportionality. In general terms, the instruction followed the second grade mathematics program, but it should be noted that the related concepts usually appear in textbooks separately under different topic headings: *proportionality*, *similarity*, *scales*, *percentages*, *linear functions*, *etc.* … hence the need to relate them within the above-mentioned context of the construction of the reference map. The structure of the instructional module was adapted from Project LEAP (Learning about Ecology, Animals and Plants, 1995). Under this approach, activities are grouped into three phases: *introduction, focusing and summary*. The process begins with the presentation of the most inclusive concepts, after which progressive differentiations and the more significant integrative reconciliations are made. The final stage is the application of the information discussed throughout the instruction period. Ideas from several published texts were used in the design of the activities for this module, which was written in the Basque language ready for presentation in the classroom.

4. The teacher did not use the same approach with all three groups when giving the instruction during the second term of the school year. 29 students from one class were designated to be the control group, and the other two classes, that is a total of 55 students, made up the study group. The innovative IM was used in the two classes that made up the study group and the chosen methodology required students to work in small groups of five during the three formal phases of the module. In the control group, the topic of proportionality was dealt with in a more traditional manner, following the sequence suggested in the text book, without any clear distinction between the introductory, focus and summary stages, and students worked individually through some of the programmed activities. The last activity in the instructional module for all the students was for each to construct his/her own final CM from the same 25 concepts used in the map prior to instruction. Figure 3 shows a post instruction map by the same author of the pre-instruction map shown earlier. It is important to note the improvement achieved by this student, the second map bearing a fairly close resemblance to the reference map produced by the teacher.

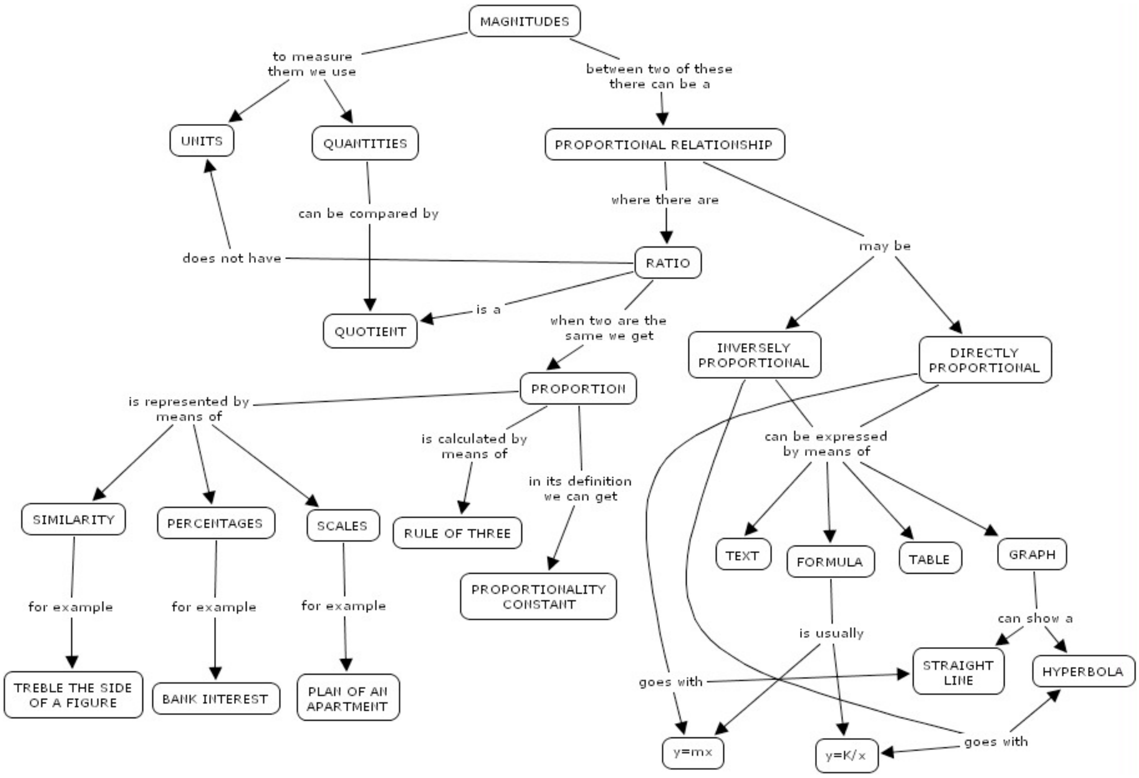

Figure 3. I. A.´s final map.

5. The series of variables defined for the comparative analysis of the CMs of all 84 students, before and after delivery of the instructional module, were based on the indicators shown in Table 1 above:

- V_1 Evidence of the student's ability to make a clear distinction between concepts and linking words and accurately represent the direction of the relationships between concepts. A qualitative variable that takes a value of YES or NO depending on the presence or absence of such evidence.
- V_2 The number of concepts used. A quantitative variable.
- V_3 Clear identification of the most inclusive concept. A qualitative variable that takes a value of YES or NO, according to whether or not the most inclusive concept is accurately identified.
- V_4 Percentage of faulty propositions relative to total number of propositions made by the student. A quantitative variable.
- V_5 Coherence in the hierarchical arrangement of concepts by level of inclusivity. A qualitative variable that takes a value of YES or NO, depending on the presence or absence of a logical hierarchical structure.
- V_6 An example of the super-ordination of an inclusive concept. A qualitative variable that takes a value of YES or NO, depending whether the map shows a relationship between the concepts of ***ratio*** and ***proportional relationship*** or not.
- V_7 Complex progressive differentiation of the more inclusive concepts. A qualitative variable with three categories: NONE if there is no presence, SOME if one inclusive concept has been differentiated or HIGH if two or more inclusive concepts have been differentiated.
- V_8 Sequences of linear relationships between concepts. A qualitative variable that takes a value of YES or NO, depending whether there are more than three linear sequences or not.
- V_9 Number of valid crosslinks. A quantitative variable.

3 Discussion and results

The SPSS (Statistical Package for the Social Sciences) was used in this research to obtain the results of the comparative analysis of the CMs of the 84 students, before and after delivery of the instructional module.

The baseline homogeneity of the treatment and control groups was tested using an equality of means test for the quantitative variables and homogeneity tests for the qualitative variables. The following table (see Table 2), which gives the corresponding means, the category percentages of the qualitative variables, and the significance level of the tests performed, confirms the baseline homogeneity of the two groups, that is, absence of significant variation in any of the variables considered.

Table 2. Baseline comparison of the treatment and control groups.

variable	**Treatment group**		**Control group**		**Significance level**
	Mean	*percentage*	*mean*	*Percentage*	
V_1 concepts and link words		YES 87.3 NO 12.7		YES 86.2 NO 13.8	NO (p=0.890)
V_2 nº of concepts $_{used}$	16.35		16.07		NO (p=0.834)
V_3 most inclusive concept		YES 85.4 NO 14.5		YES 75.9 NO 24.1	NO (p=0.275)
V_4 faulty propositions	53.15		54.59		NO (p=0.777)
V_5 coherent hierarchical structure		NO 100		NO 100	NO
V_6 superordination		NO 100		NO 100	NO
V_7 progressive differentation		NONE 100		NONE 100	NO
V_{81} linear sequences		YES 18.2 NO 81.8		YES 17.2 NO 82.8	NO (p=0.915)
V_9 nº cross links	0		0		NO

The following table (see Table 3) describes the baseline and final performance of the treatment group, showing the corresponding means of the quantitative variables, the category percentages of the qualitative variables, and the significance level of the tests performed:

Table 3. Comparison of baseline and final performance of treatment group.

variable	**Baseline**		**Final**		**Significance level**
	Mean	*percentage*	*mean*	*percentage*	
V_1 concepts and link words		YES 87.3 NO 12.7		YES 85.4 NO 14.5	
V_2 nº of concepts used	16.35		22.58		YES. Highly significant (p=0.000)
V_3 most inclusive concept		YES 85.4 NO 14.5		YES 92.7 NO 7.3	
V_4 faulty propositions	53.15		18.67		YES. Highly significant (p=0.000)
V_5 Coherent hierarchical structure		NO 100		YES 29 NO 71	
V_6 superordination		NO 100		YES 41.8 NO 58.2	
V_7 progressive differentiation		NONE 100		HIGH 9.1 SOME 27.3 NONE 63.6	
V_8 linear sequences		YES 18.2 NO 81.8		YES 14.5 NO 85.5	
V_9 nº of cross links	0		1.04		YES. Highly significant (p=0.000)

It is possible to reject the presence of significant variation with respect to the first of the variables considered, that is, the presence of a clear distinction between concepts and link words and an indication of the direction of the relationships between concepts is similar for both observations. The remaining variables nevertheless show significant differences between the baseline and final observations, revealing a clearly positive evolution in the students of the treatment group in the following terms:

- An increase in the number of concepts used in the final maps.
- A greater number of students have clearly identified the most inclusive concept in the final maps.
- A reduction in the percentage share of faulty propositions to total propositions in the final maps.
- The presence in some cases of a coherent hierarchical structure in terms of the inclusivity of the concepts in the final maps.

- The presence in some cases of an example of super-ordination of an inclusive concept, such that some students represent the relationship between the concepts of ***ratio*** and ***proportional relationship*** in their final maps.
- The presence in the final maps of some of the students of complex progressive differentiation of the more inclusive concepts.
- Less presence of linear sequences of relationships between concepts in the final maps.
- An increase in the number of crosslinks in the final maps.

The following table (see Table 4) depicts a final comparison of the study variables between the treatment and control groups. Like the tables above, it shows the corresponding means of the quantitative variables, the category percentages of the qualitative variables, and the significance level of tests performed:

Table 4. Final comparison of the treatment and control groups.

variable	**Treatment group**		**Control group**		**Significance level**
	mean	*percentage*	*mean*	*percentage*	
V_1 concepts and link words		YES 85.4 NO 14.5		YES 93.1 NO 6.9	NO (p=0.303)
V_2 nº of concepts used	22.58		20.72		YES (p=0.022)
V_3 most inclusive concept		YES 92.7 NO 7.3		YES 65.5 NO 34.5	YES (p=0.001)
V_4 faulty propositions	18.67		34.93		YES (p=0.000)
V_5 coherent hierarchical structure		YES 29 NO 71		NO 100	YES (p=0.001)
V_6 superordination		YES 41.8 NO 58.2		NO 100	YES (p=0.000)
V_7 progessive differentiation		HIGH 9.1 SOME 27.3 NONE 63.6		NONE 100	YES (p=0.001)
V_8 linear sequences		YES 14.5 NO 85.5		YES 31 NO 69	YES (p=0.074)
V_9 nº cross links	1.04		0.21		YES (p=0.021)

Again, no significant differences emerge with respect to the first of the variables considered, whereas in terms of the remaining variables the two groups differ significantly by the end of the experiment.

4 Conclusions

The results of the statistical testing of the ML indicators by SPSS reveal statistically significant differences between members of the treatment group and members of the control group in terms of indicators of ML. It is also possible to observe a clearly positive evolution in the students of the treatment group, who report significant differences between the baseline and final observations in terms of an increase in the number of concepts used, a reduction in the percentage share of faulty to total propositions and an increase in the number of cross links, an improvement in the clear identification of the most inclusive concept, a clearer hierarchical arrangement of the concepts coherent with their degree of inclusivity, a reduction in the number of linear sequences and an increase in the number of progressive differentiations integratively reconciled. The only aspect in which no significant difference can be observed is in the first of the variables considered. In other words, both groups are similar at the baseline and at the end of the experiment as far as clarity in the differentiation between concepts and link words and the direction of the relationships between concepts are concerned. This shows that the instructional process was not effective in this respect.

These results, together with the evaluation of the evolution of the students' CMs, show that the delivery in a standard school setting of an innovative IM based on the promotion of positive attitudes, detection of students' initial beliefs and the use of conceptually transparent teaching material, has proved highly successful in helping the treatment group to achieve more meaningful learning on the topic of proportionality. If possible, the experiment would be worth repeating with larger samples.

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